Name _____ Period _____

6.3C *Finding All Solutions of Polynomial Equations Algebraically*

1. Find all the zeros of the polynomial function. At least one of the zeros has been provided. (Note - some of the zeros are irrational numbers and cannot be expressed as fractions or decimals and some of the zeros are imaginary numbers and cannot be seen on the graph). NOTE - all answers MUST be exact – no approximate answers!

a) $y = 2x^3 + 14x^2 + 19x - 2$

$x = -2$

$$\begin{array}{r|rrr} -2 & 2 & 14 & 19 & -2 \\ & & -4 & -20 & 2 \\ \hline & 2 & 10 & -1 & 0 \end{array}$$

$2x^2 + 10x - 1 = 0$

$x = \dfrac{-10 \pm \sqrt{(10)^2 - 4(2)(-1)}}{2(2)}$

$= \dfrac{-10 \pm \sqrt{100+8}}{4} = \dfrac{-10 \pm \sqrt{108}}{4}$

$= \dfrac{-10 \pm 6\sqrt{3}}{4} = \dfrac{-5 \pm 3\sqrt{3}}{2}$

$x = -2, \ \dfrac{-5 \pm 3\sqrt{3}}{2}$

b) $f(x) = x^3 - x^2 - 12x + 90$

$x = -5$

$$\begin{array}{r|rrrr} -5 & 1 & -1 & -12 & 90 \\ & & -5 & 30 & -90 \\ \hline & 1 & -6 & 18 & 0 \end{array}$$

$x^2 - 6x + 18 = 0$

$x^2 - 6x + 9 = -18 + 9$

$\sqrt{(x-3)^2} = \sqrt{-9}$

$|x-3| = 3i$

$x = 3 \pm 3i$

$x = -5, \ 3 \pm 3i$

c) $y = x^4 - 3x^3 - 20x^2 + 50x$

$x = 0, \ x = 5$

$y = x(x^3 - 3x^2 - 20x + 50)$

$$\begin{array}{r|rrr|r} 5 & 1 & -3 & -20 & 50 \\ & & 5 & 10 & -50 \\ \hline & 1 & 2 & -10 & 0 \end{array}$$

$x^2 + 2x - 10 = 0$

$x^2 + 2x + 1 = 10 + 1$

$\sqrt{(x+1)^2} = \sqrt{11}$

$|x+1| = \sqrt{11}$

$x = -1 \pm \sqrt{11}$

$x = 0, 5, -1 \pm \sqrt{11}$

2. Find all the solutions of the polynomial equation. At least one of the factors has been provided. (Note - some of the solutions are irrational numbers and cannot be expressed as fractions or decimals and some of the zeros are imaginary numbers and cannot be seen on the graph). NOTE - all answers MUST be exact - no approximate answers!

a) $x^3 - 7x^2 - 22x + 160 = 0$

$(x-5)$

$$\begin{array}{r|rrrr} 5 & 1 & -7 & -22 & 160 \\ & & 5 & -10 & -160 \\ \hline & 1 & -2 & -32 & 0 \end{array}$$

$x^2 - 2x - 32 = 0$

$x^2 - 2x + 1 = 32 + 1$

$\sqrt{(x-1)^2} = \sqrt{33}$

$|x-1| = \sqrt{33}$

$x = 1 \pm \sqrt{33}$

$x = 5, \ 1 \pm \sqrt{33}$

b) $3x^3 + 19x^2 - 3x - 3 = 0$

$(3x+1)$

$$\begin{array}{r|rrrr} -\frac{1}{3} & 3 & 19 & -3 & -3 \\ & & -1 & -6 & 3 \\ \hline & 3 & 18 & -9 & 0 \end{array}$$

$\dfrac{3x^2 + 18x - 9}{3} = \dfrac{0}{3}$

$x^2 + 6x - 3 = 0$

$x^2 + 6x + 9 = 3 + 9$

$\sqrt{(x+3)^2} = \sqrt{12}$

$|x+3| = 2\sqrt{3}$

$x = -3 \pm 2\sqrt{3}$

$x = -\frac{1}{3}, \ -3 \pm 2\sqrt{3}$

c) $x^4 + 9x^3 + 36x^2 + 54x = 0$

x and $(x+3)$

$x(x^3 + 9x^2 + 36x + 54) = 0$

$x = 0, \ -3$

$$\begin{array}{r|rrrr} -3 & 1 & 9 & 36 & 54 \\ & & -3 & -18 & -54 \\ \hline & 1 & 6 & 18 & 0 \end{array}$$

$x^2 + 6x + 18 = 0$

$x^2 + 6x + 9 = -18 + 9$

$\sqrt{(x+3)^2} = \sqrt{-9}$

$|x+3| = 3i$

$x = -3 \pm 3i$

$x = 0, \ -3, \ -3 \pm 3i$

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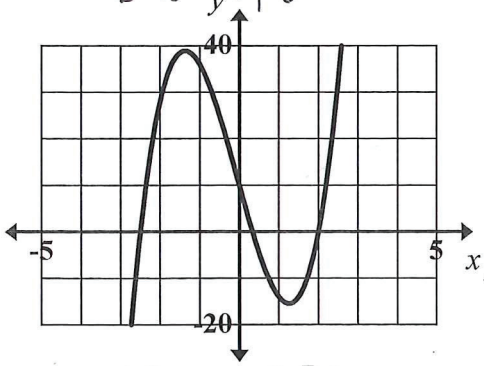
6.3C *Finding All Solutions of Polynomial Equations Algebraically*

3. Find all the x-intercepts of the polynomial function. Give exact answers. (Note - some of the x-intercepts are irrational numbers and cannot be expressed as fractions or decimals). The graph of the function is provided. You may use your graphing calculator to obtain a better graph if you like. NOTE - all answers MUST be exact - no approximate answers!

a) $y = 6x^3 + x^2 - 31x + 10$

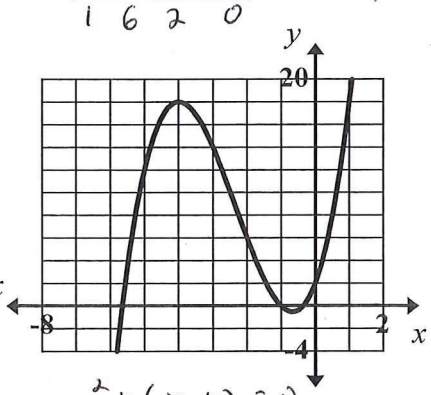


$6x^3 + 13x - 5 = 0$

$(3x-1)(2x+5) = 0$

$x = \frac{1}{3}, -\frac{5}{2}$

b) $y = x^3 + 7x^2 + 8x + 2$



$x^2 + 6x + 2 = 0$

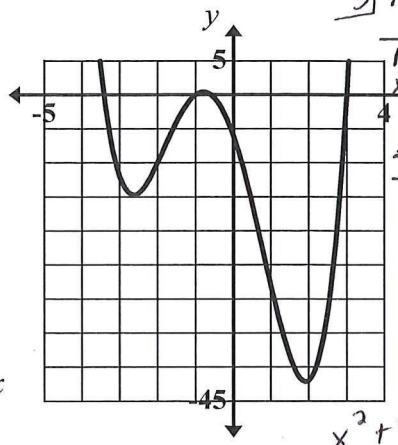
$x^2 + 6x + 9 = -2 + 9$

$\sqrt{(x+3)^2} = \sqrt{7}$

$|x+3| = \sqrt{7}$

$x = -1, -3 \pm \sqrt{7}$

c) $y = x^4 + 2x^3 - 9x^2 - 16x - 6$



$x^3 + 5x^2 + 6x + 2$

$x^2 + 4x + 2 = 0$

$x^2 + 4x + 4 = -2 + 4$

$\sqrt{(x+2)^2} = \sqrt{2}$

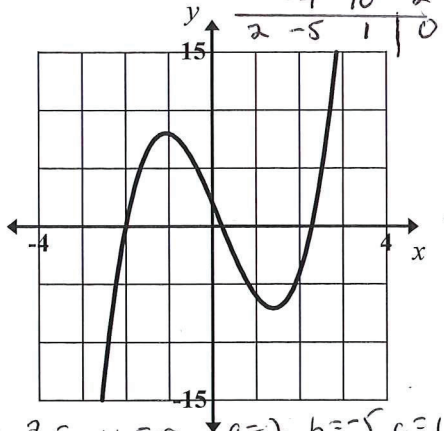
$|x+2| = \sqrt{2}$

$x = -2 \pm \sqrt{2}$

$x = 3, -1, -2 \pm \sqrt{2}$

4. Find all the zeros of the polynomial function. Give exact answers. (Note - some of the zeros are irrational numbers and cannot be expressed as fractions or decimals and some of the zeros are imaginary numbers and cannot be seen on the graph). The graph of the function is provided. . You may use your graphing calculator to obtain a better graph if you like. NOTE - all answers MUST be exact - no decimals!

a) $f(x) = 2x^3 - x^2 - 9x + 2$



$2x^2 - 5x + 1 = 0$

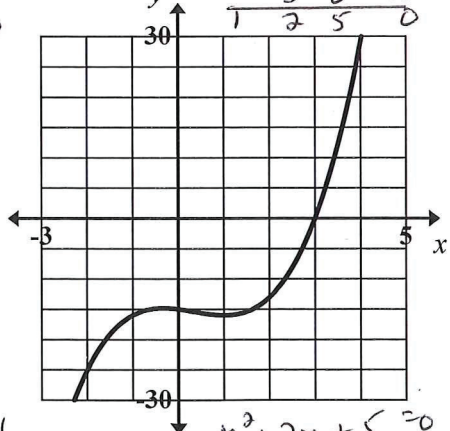
$a=2, b=-5, c=1$

$x = \frac{5 \pm \sqrt{(-5)^2 - 4(2)(1)}}{2(2)}$

$= \frac{5 \pm \sqrt{25-8}}{4}$

$x = -2, \frac{5 \pm \sqrt{17}}{4}$

b) $g(x) = x^3 - x^2 - x - 15$



$x^2 + 2x + 5 = 0$

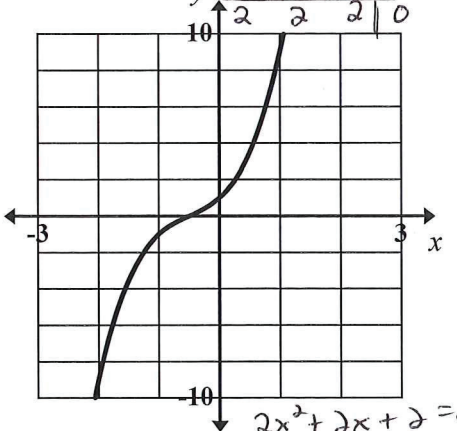
$x^2 + 2x + 1 = -5 + 1$

$\sqrt{(x+1)^2} = \sqrt{-4}$

$|x+1| = 2i$

$x = 3, -1 \pm 2i$

c) $h(x) = 2x^3 + 3x^2 + 3x + 1$



$2x^2 + 2x + 2 = 0$

$x^2 + x + 1 = 0$

$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)}$

$= \frac{-1 \pm \sqrt{-3}}{2}$

$x = -\frac{1}{2}, \frac{-1 \pm i\sqrt{3}}{2}$

Name _____ Period _____

6.3C *Finding All Solutions of Polynomial Equations Algebraically*

5. Find all the solutions to the polynomial equation $f(x) = 0$. Give exact answers. (Note - some of the solutions are irrational numbers and cannot be expressed as fractions or decimals and some of the zeros are imaginary numbers and cannot be seen on the graph).

a) $f(x) = x^3 + x^2 - 5x - 2$

2 | 1 1 -5 -2
2 6 2
1 3 1 0

$x^2 + 3x + 1 = 0$

$x = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(1)}}{2(1)}$

$x = \frac{-3 \pm \sqrt{5}}{2}$

$x = 2, \frac{-3 \pm \sqrt{5}}{2}$

b) $f(x) = x^4 - 4x^3 + 4x^2 - 64$

X-ints at -2 and 4

-2 | 1 -4 4 0 -64
-2 12 -32 64
1 -6 16 -32 | 0

$x^3 - 6x^2 + 16x - 32 = 0$

4 | 1 -6 16 -32
4 -8 32
1 -2 8 | 0

$x^2 - 2x + 8 = 0$

$x^2 - 2x + 1 = -8 + 1$

$\sqrt{(x-1)^2} = \sqrt{-7}$

$|x - 1| = i\sqrt{7}$

$x = 1 \pm i\sqrt{7}$

$x = -2, 4, 1 \pm i\sqrt{7}$

c) $f(x) = x^4 - 2x^3 - 14x^2 + 30x + 9$

* Double zero at $x = 3$

3 | 1 -2 -14 30 9
3 3 -33 -9
1 1 -11 -3 | 0

3 | 1 1 -11 -3
3 12 3
1 4 1 | 0

$x^2 + 4x + 1 = 0$

$x^2 + 4x + 4 = -1 + 4$

$\sqrt{(x+2)^2} = \sqrt{3}$

$|x + 2| = \sqrt{3}$

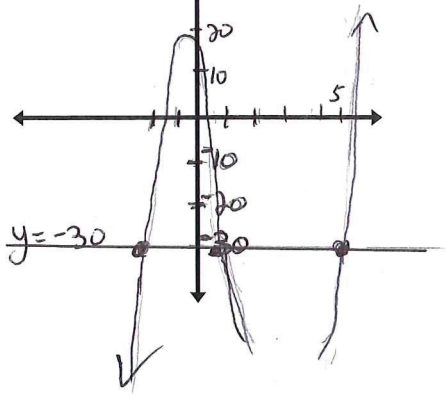
$x = -2 \pm \sqrt{3}$

$x = 3^*, -2 \pm \sqrt{3}$

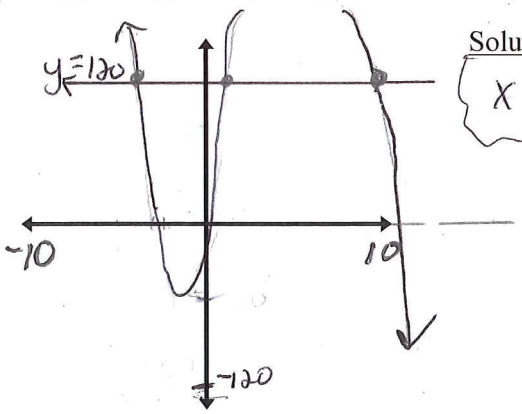
6. Use a graphing utility to identify the solution(s) to each equation. Include a sketch of the graph that appeared. You will need to make adjustments to the window to see all the solutions. Find ALL solutions (real and imaginary) to these equations. All solutions must be exact – no approximate answers.

a) $4x^3 - 15x^2 - 31x = -30$

$x = 5, -2, 0.75$ ← Solutions



b) $-2x^3 + 15x^2 + 62x = 120$



Solutions

$x = -4, 1.5, 10$

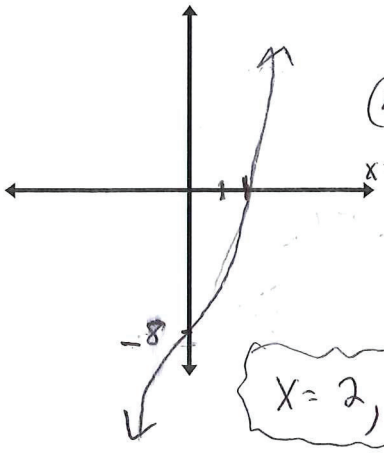
Name _____ Period _____

6.3C *Finding All Solutions of Polynomial Equations Algebraically*

6. (continued) Use a graphing utility to identify the solution(s) to each equation. Include a sketch of the graph that appeared. You will need to make adjustments to the window to see all the solutions. Find ALL solutions (real and imaginary) to these equations. All solutions must be exact – no approximate answers.

c) $2x^3 - 16 = 0$

$2(x^3 - 8) = 0$



Solutions

$(x)^3 - (2)^3 = 0$

$(x-2)(x^2+2x+4) = 0$

$x = 2$, $x^2+2x+1 = -4+1$

$\sqrt{(x+1)^2} = \sqrt{-3}$

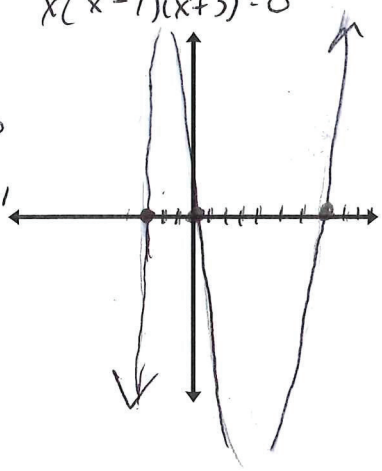
$|x+1| = i\sqrt{3}$

$x = -1 \pm i\sqrt{3}$

$x = 2, -1 \pm i\sqrt{3}$

d) $x(x^2 - 4x - 21) = 0$

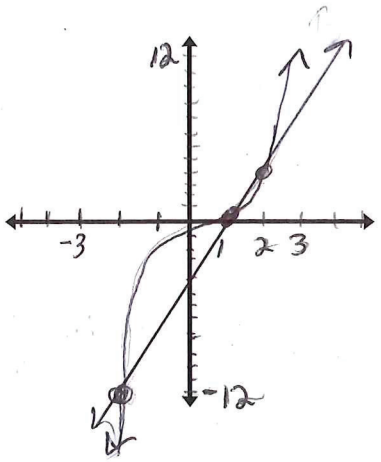
$x(x-7)(x+3) = 0$



Solutions

$x = -3, 0, 7$

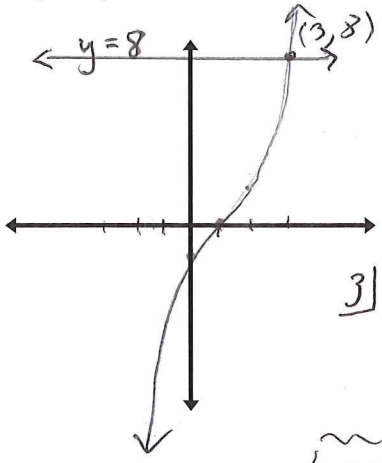
e) $x^3 - x^2 = 4x - 4$



Solutions

$x = -2, 1, 2$

f) $(x-1)^3 = 8$



Solutions

$(x-1)(x-1)(x-1) = 8$

$(x-1)(x^2-2x+1)$

$x^3 - 3x^2 + 3x - 1 - 8 = 0$

$x^3 - 3x^2 + 3x - 9 = 0$

3 | 1 -3 3 -9

3 0 9

1 0 3 0

$x^2 + 3 = 0$

$\sqrt{x^2} = \sqrt{-3}$

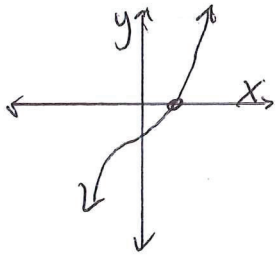
$x = \pm i\sqrt{3}$

$x = 3, \pm i\sqrt{3}$

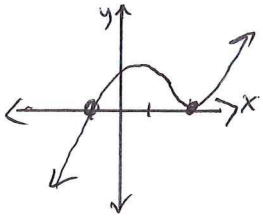
7. How many solutions can a cubic function have? Explain your answer clearly and ~~give an example of each~~.

A cubic function always has 3 solutions. Either:

1 real & 2 imaginary solutions,

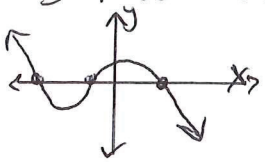


2 different real solutions but one of them is a duplicate solution where the graph just touches the x-axis, like



$x = -1, 2, 2$ but written $x = -1, 2$

or 3 real solutions



Section 6.3C